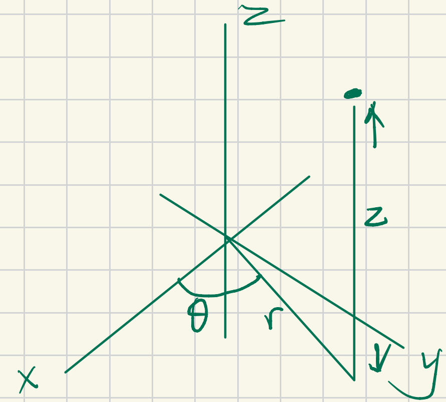
1.4

Cylindrical and Spherical coordinates

We learn

- polar coordinates in 2-D (review) (r, θ)
- cylindrical coordinates in 3-D
- Spherical coordinates in 3-D.

Cylindrical coordinates (r, θ, z)



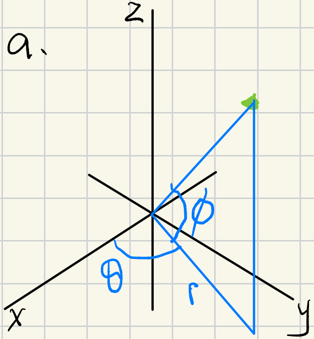
Example:
Find the ~~cylindrical~~ coordinates of (x,y,z) = (1,-1,1)

Soln: $\left(\sqrt{2}, \frac{7\pi}{4}, 1\right)$

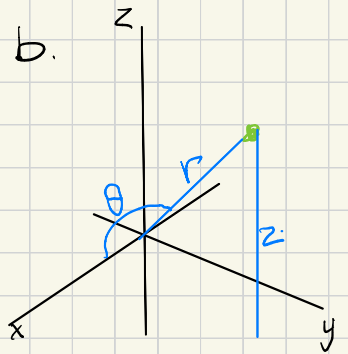
Pre-class Warm-up!!!!

Which picture best describes cylindrical coordinates for R^3 ?

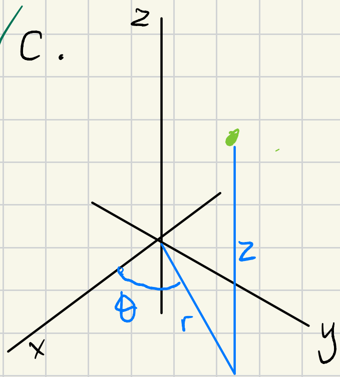
a.



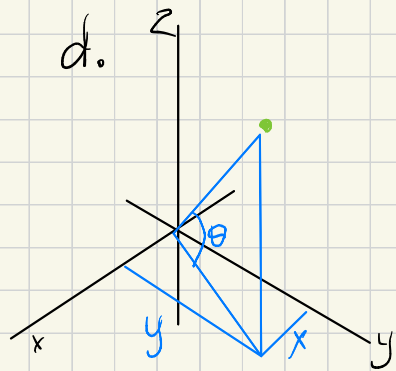
b.



✓ c.



d.

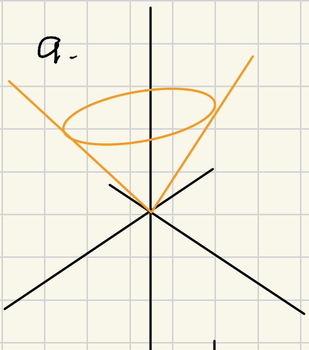


What are the surfaces

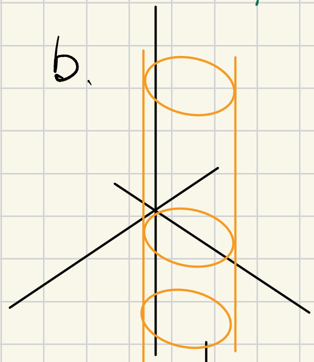
- z^2 = r^2 e. $z = \pm r$
- z = r^2 c.
- r = 4 sin theta $= 4\sin\theta$ b. eg. $\theta = 0 \Rightarrow r = 0$

What is the region
r in [0,1], theta in [0,π], z in [-1,1] ? f.

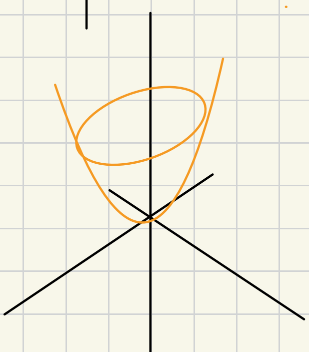
a.



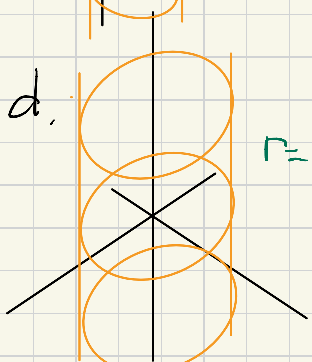
b.



c.

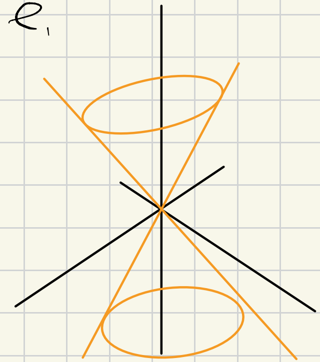


d.

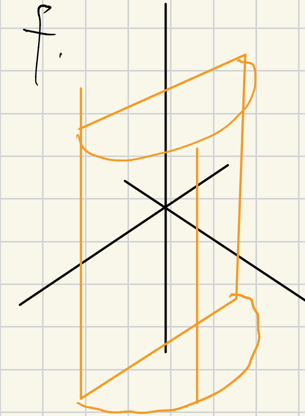


$r = 6$

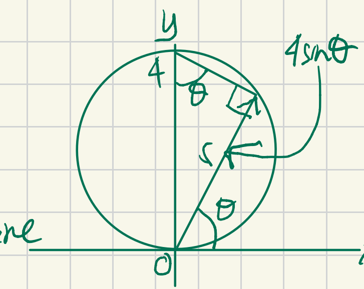
e.



f.



$r = 4\sin\theta$
is a circle
in the x-y plane



Spherical coordinates

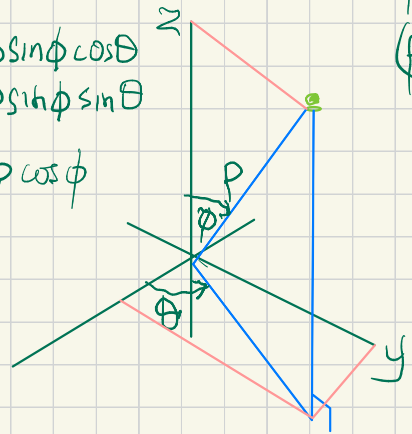
(ρ, θ, ϕ) in the book

(ρ, ϕ, θ) elsewhere

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$



Usually $\rho \geq 0$

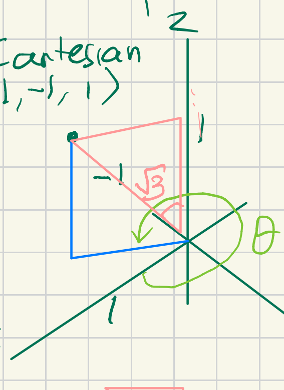
$0 \leq \theta \leq 2\pi$

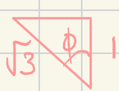
$0 \leq \phi \leq \pi$

Example

Cartesian

$(1,-1,1)$





$$\theta = \frac{7\pi}{4}$$

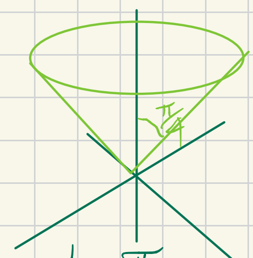
$$\rho = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

$$\cos\phi = \frac{1}{\sqrt{3}}, \quad \phi = \cos^{-1}\frac{1}{\sqrt{3}}$$

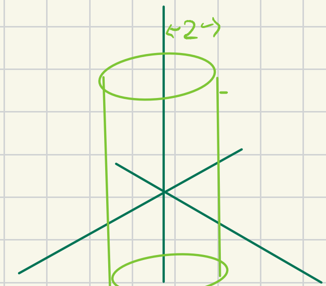
$$(\rho, \theta, \phi) = \left(\sqrt{3}, \frac{7\pi}{4}, \cos^{-1}\frac{1}{\sqrt{3}}\right)$$

Examples:

- Put (x,y,z) = (1,-1,1) in spherical polar coordinates
- Sketch the surface p = 2 $\rho = 2$
  This a sphere radius 2, center the origin
- Sketch surface phi = π / 4
- Sketch the surface rho sin phi = 2



$\phi = \pi/4$



$\rho \sin\phi$ = distance to z axis